Measuring and Explaining Inequality

Peter Warr*

This paper is intended as methodological background for the measurement and explanation of changes in inequality in the East Asian economies covered in the study. Over the last 20 years or so the changes in inequality in these economies have been quite different, so there is very good material for a useful comparative study.

1. Overview

The theme of the discussion below is that the measurement of inequality must be consistent with our attempts to explain it. For reasons which will become apparent, it is proposed that the measurement of inequality focus on incomes rather than expenditures. It is also proposed that the class of Generalized Entropy measures of inequality be included in the measures used. The proposed method of explaining inequality is through decomposition of levels of income inequality and changes in it into its components. This proposed decomposition takes two forms:

(i) *horizontal* decomposition into inequality within and between social groups; and

(ii) *vertical* decomposition into the contributions of the various sources of incomes.

Each of these forms of decompositions can be conducted in *static* form – decomposing the cross-sectional level of inequality at a particular point in time – and in *dynamic* form – explaining the changes in inequality over time.

The measurement of income should be capable of supporting decompositions of all of the above forms.

2. Measuring inequality

We are interested ultimately in explaining the pattern of inequality and changes in it. The kinds of explanations we have available relate mostly to *incomes*. This strongly suggests that the measures of inequality that we use in the study should focus on incomes as well, rather than expenditures. Some statistical agencies do focus on incomes for the measurement of inequality and poverty incidence (e.g. Thailand) while others use expenditures (e.g. Indonesia). In addition, the definition of 'income' is not the same across countries, and the same problem would arise if we were focusing on expenditures. For example, some statistical agencies include imputed rent on owner-occupied housing as a source of income and a component of expenditures (Thailand's Socio-economic Survey is an example). Others do not.

It is clearly desirable to arrive at measures of inequality which are comparable across countries and this requires reaching agreement, for the purposes of our study, on definitions of what is included in income. Unfortunately, in the case of some case countries the data which are available to independent researchers do not support that effort. As the chapter on Malaysia will show, Malaysia is an example of this kind of problem. In those cases, the important thing is to make it clear what the data do and

* Australian National University

do not measure, so as to minimize confusion with results for countries where the data are assembled differently.

Desirable features of inequality measures

The following five axioms have been advanced as desirable features of a measure of inequality:

(a) *The transfer principle:* if income is hypothetically transferred from a poorer to a richer person, without reversing that ranking, then the inequality measure should rise (at least not fall).
(b) *Scale independence:* if all incomes change by a uniform proportion, the inequality measure should not change.
(c) *Population independence:* if two identical populations are merged, the inequality measure should not change.
(d) *Anonymity:* The inequality measure must be independent of any characteristic of individuals other than their income.
(e) *Decomposability:* It should be possible to decompose the measure such that the total measure of inequality is equal to the sum of inequality within sub-groups and between sub-groups.

Cowell (1995) shows that any measure which satisfies all five of the above axioms belongs to the class of Generalized Entropy (GE) measures, which have the form

$$GE(\alpha) = \frac{1}{\alpha(\alpha-1)}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\bar{y}}\right)^{\alpha} - 1\right] \qquad (1)$$

where n is the number of individuals in the sample, y_i is the income of individual i, $\bar{y}$ is the arithmetic mean of all incomes, and α is a parameter. The range of *GE* is from zero to infinity, with zero corresponding to complete inequality. It should be noted that an income of zero for any individual in the data set, will mean that the measured value of *GE(0)* will be infinite.

As the parameter α increases, the *GE* measures become less sensitive to inequality at the lower end of the distribution and more sensitive to inequality at the upper end. Three special cases are of special interest, corresponding to values of α of 0, 1 and 2.

$$GE(0) = \frac{1}{n}\sum_{i=1}^{n}\log\frac{\bar{y}}{y_i} \qquad (2)$$

$$GE(1) = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{\bar{y}}\log\frac{y_i}{\bar{y}} \qquad (3)$$

These cases correspond to the mean log deviation (α=0) and the Theil index of inequality (α=1). When α=2, the GE measure corresponds to one half the squared coefficient of variation, *CV*, where

$$CV = \frac{1}{\bar{y}}\left[\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2\right]^{1/2} \qquad (4)$$

GE(0) applies more weight to differences in incomes at the lower tail of the distribution,
GE(1) applies equal weight to all values in the distribution and *GE*(2) applies more weight at the upper end.

The Gini coefficient and the Atkinson class of inequality measures do not satisfy the decomposition axiom and thus do not belong to the GE class of measures. The only case in which the Gini coefficient is consistent with the decomposition axiom is when the groups considered are non-overlapping in their incomes, a very restrictive condition. On the one hand, this means that their suitability for this study is limited by the fact that they cannot be properly decomposed. On the other hand, the fact that these measures are well known means that their calculation in this study is helpful for comparison with other studies dealing with inequality.

There is nothing to prevent us from including these measures (Gini and Atkinson) in the study, and I think we should, at least in the case of the Gini coefficient, but I think we must also calculate GE measures because they lend themselves to decomposition. Fortunately, the standard statistical packages make this possible. An example is STATA, a widely used econometric package and I will be referring to it below.[1]

3. Static decompositions

There are two kinds of decompositions that we wish to achieve to describe the sources of inequality at a particular point in time.

(a) A decomposition of total inequality for the whole population into inequality *between* and *within* major groups. The groups that may be of interest may include geographical regions, occupational categories, ethnic categories, educational attainment categories, age groups, family sizes, and so forth.

(b) A decomposition of the inequality of total income for the whole population or any sub-group into the contributions to total inequality which arise from the various *components of total income*, such as labour income (skilled and unskilled), income from land, income from ownership of capital, remittance income, and so forth. Further breakdown of these categories may be desirable.

The purpose of the decomposition is to see what produced the observed changes in total income inequality. Each country author is then in a much-improved position to attempt to explain, in the context of his or her country, why these changes occurred.

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[1] Another package which computes all of the measures described here is DAD and is described in Duclos, Araar and Fortin (2002). This package has the advantage of being downloadable from the web site: http://www.mimap.ecn.ulaval.ca/.

Decomposition by population sub-groups
For reasons that will become apparent below, we shall refer to this as horizontal decomposition of inequality. 'Within group' inequality, $GE_W(\alpha)$, is measured from

$$GE_W(\alpha) = \sum_{j=1}^{K} g_j GE(\alpha)_j , \qquad (5)$$

where $GE(\alpha)_j$ is simply the value of the $GE(\alpha)$ measure applied to sub-group j and the sub-group weights g_j are given by

$$g_j = v_j^{\alpha} w_j^{1-\alpha} , \qquad (6)$$

where v_j and w_j are the income shares and the population shares, respectively of the sub-groups $j = 1,2,...K$. Clearly, when (α=0), the weights are just population shares and when (α=1) they are just income shares. When (α=2), the group weights are the square of the income share divided by the population share.

The measure of 'between group' inequality, $GE_B(\alpha)$, involves re-estimating total inequality when each member of each sub-group *j* is given the mean income for that sub-group, $\bar{y}_j$, and then calculating

$$GE_B(\alpha) = \frac{1}{\alpha(\alpha-1)}\left[\frac{1}{n}\sum_{j=1}^{K}\left(\frac{\bar{y}_j}{\bar{y}}\right)^{\alpha} - 1\right], \qquad (7)$$

where $\bar{y}$ is the mean income for the whole population. It is then true that

$$GE(\alpha) = GE_W(\alpha) + GE_B(\alpha) . \qquad (8)$$

Once these calculations have been performed, a simple measure of the proportion of total inequality that takes the form of between group inequality is (Cowell and Jenkins 1995)

$$H_B(\alpha) = GE_B(\alpha) / GE(\alpha) \qquad (9)$$

This is a very intuitive measure. It tells us the proportion of total inequality (using measure $GE(\alpha)$) that takes the form of (is 'explained' by) inequality between geographical regions, educational categories, occupational categories, age groups, and do forth. The remainder of the total inequality takes the form of (is 'explained' by) inequality within these various categories. For instance, we may be particularly interested to know how much of the total inequality of incomes takes the form of inequality between the major regions.

It is important to note that the value of this measure is not independent of the particular member of the $GE(\alpha)$ class that is chosen (i.e. it is not independent of α).

Decompositions of this kind are supported by the STATA program under the set of commands **ineqdeco**. Unit record data (i.e. household level observations) are required. This set of sub-programs estimates the levels of the GE class of measures of inequality, the Atkinson measure, the Gini coefficient and percentile share ratios, and then facilitates decomposition of the GE class.

Decomposition by source of income

Because total income is made up of streams of income originating in different ways (labour income, income from ownership of land, income from capital, and so forth) it is of interest to know not only the contributions of the various components to mean income, but also how much of the total inequality in total incomes arises from inequality across individuals in these various income sources. It will be apparent that this is quite a different exercise from a decomposition of inequality in total incomes across social groups, and for that reason it is called here vertical inequality.

Let total income, *y*, consist of income from various sources, *s*, such that

$$y = \sum_{s=1}^{S} y_s \,. \qquad (10)$$

We wish to achieve a decomposition of total inequality, *I*, such that

$$I = \sum_{s=1}^{S} I_s \,. \qquad (11)$$

The proportion of total inequality that derives from source *s* is then simply

$$i_s = I_s / I \,. \qquad (12)$$

and $\sum_{s=1}^{S} i_s = 1$.

The exact decomposition formula depends on the measure of inequality that is used, but Shorrocks (1982) shows that there is a unique way of decomposing income inequality (or inequality in any additive variable) in this way, which takes the form

$$i_s = \rho_s \sigma_s / \sigma = \mathrm{cov}(y_s, y) / \sigma^2 \qquad (13)$$

where ρ_s is the correlation coefficient between income from source *s,* y_s, and total income, *y*, σ_s is the standard deviation of y_s and σ is the standard deviation of *y,* $\mathrm{cov}(y_s, y)$ is the covariance of y_s and *y, and* σ^2 is the variance of *y*.. The striking feature of Shorrocks' result is that (13) applies for *any* measure of inequality which satisfies some innocuous axioms that Shorrocks specifies. This includes the Atkinson and Gini measures, as well as the GE class of measures.

In the case of the *GE*(2) measure, which is mathematically the simplest, the fact that, as stated above, this measure reduces to one half the squared coefficient of variation, *CV*, implies that

$$I_s = i_s GE(2) = \rho_s f_s \sqrt{GE(2).GE(2)_s} \qquad (14)$$

calculated across all individuals, $f_s = \bar{y}_s / \bar{y}$ is the mean income from source *s* divided by the mean of total income, $GE(2)_s$ is the value of the GE(2) measure applied to the distribution of total incomes, *y*, and $GE(2)_s$ is the value of the GE(2) measure applied to the distribution of incomes from source *s*, y_s.

An important practical consideration is that we require a measure of inequality that is defined for zero incomes because there will always be individuals in the sample whose income from some source or other is zero. This rules out the *GE*(0) measure. This measure should be avoided whenever zero incomes or income components are a likely feature of the data set to be used. Otherwise, arbitrary devices must be used to avoid infinite values of the inequality measure preventing clear messages from being extracted from the data.

It should be noted that although the sum of the contributions of all income sources to total inequality must be positive, the contribution of a *particular* income source *s* to total inequality can be positive or negative. For example, if there was a source of income that favoured the poor, in that they received a higher share of total income from this source than the rich, whereas they receive (by definition) a lower share of total income from all sources than the rich, the value of I_s from this source would be negative – its contribution to the distribution of total incomes is equalizing rather than disequalizing.

Decompositions of total income inequality by source of income are supported by the STATA program under the set of commands **ineqfac**. Unit record data (i.e. household level observations) are again required, unless we wish to know the degree to which inequality *between* sub-groups arises from variations in incomes from different sources. In this case the units of observation become the incomes from the various sources for the individual sub-groups.

4. Dynamic decompositions

How can we decompose *changes* in inequality over time? As above, these issues of decomposition of changes in inequality, here referred to as *dynamic* decompositions, can be conducted in horizontal and vertical forms.

Decomposition by population sub-groups
A dynamic decomposition again involves between and within group components, but the between group component now has two parts: one due to changes in the mean incomes of the groups and another to changes in the sizes of the groups. The resulting three components of the decomposition of changes in inequality are called by Mookerjee and Shorrocks (1982):

(i) an "income effect", due to changes in relative mean incomes between the sub-groups,
(ii) an "allocation effect", due to changes in the sizes of the sub-groups, and
(iii) a "pure inequality effect", due to changes in inequality within sub-groups.

Decompositions along these lines are supported for the GE class of by the STATA package, using the **ineqdeco** set of commands.

Decomposition by source of income
Returning to the GE(2) measure, Jenkins (1995) shows that

$$\Delta GE(2) = GE(2)_t - GE(2)_{t-1} = \sum_{s=1}^{S} \Delta I_s = \sum_{s=1}^{S} \Delta[\rho_s f_s \sqrt{GE(2).GE(2)_s}\,] \qquad (15)$$

The proportionate change in GE(2) is thus

$$\Delta GE(2)/GE(2) = \sum_{s=1}^{S} i_s \Delta I_s / I_s = \sum_{s=1}^{S} \{ i_s \Delta[\rho_s f_s \sqrt{GE(2).GE(2)_s}\,]/GE(2)_s \} \qquad (16)$$

STATA also supports these decompositions, using the **ineqfac** set of commands.

5. Regression-based decompositions

Fields (2001) has proposed a useful regression-based decomposition technique which turns out to be an application of Shorrocks' result above. Suppose we have estimated income-generating equations which 'explain' the logarithm of incomes in the form

$$\ln(y^i) = a + \sum_{j=1}^{J} b_j x_j^i + u^i, \qquad (17)$$

where y^i is the income of individual i, x_j^i is the value of explanatory factor j for individual such as education, region of residence, family size, age composition of the household and so forth, *a* and the *b*'s are parameters and *u* is an error term.

In the labour economics literature, these equations are also called "earnings functions" and "wage equations". Hundreds of studies have estimated equations like these. Fields' method provides a way of using these equations to decompose the variation in inequality in terms of these same explanatory variables. Moreover, the method does not depend on the particular measure of inequality that is being used.

Fields method may be understood intuititively as follows. If we view the elements of the right hand side of (17), it is clear that we have an additive identity analagous to the sources of income identity given by (10). Shorrocks' results may now be applied directly, except that one of the "sources" appearing in (17) is the error term. This gives a decomposition of the inequality in $\ln(y^i)$ which has $J+1$ components. The first J components are the explanatory variables and the final component is the error term. Thus the decomposition of the inequality in the logarithm of incomes will have an unexplained residual component arising from this error term.

More formally, write $\alpha^i = (a, b_1^i, b_2^i, ...b_J^i, 1)$ and $z^i = (1, x_1^i, x_2^i, ...x_J^i, u^i)$. Equation (17) now has the form

$$\ln(y^i) = \sum_{r=1}^{J+2} \alpha_r z_r \qquad (18)$$

which is identical to (10). A short-cut to Fields result is now possible. If we define

$$i_r = \alpha_r \sigma_r \rho_r / \sigma = \text{cov}(\alpha_r z_r, \ln(y)) / \sigma^2 \qquad (19)$$

then from Shorrocks it follows that

$$\sum_{r=1}^{J+2} i_r = 1 \qquad (20)$$

and, remarkably, if we add only the first $J+1$ elements of this summation (thus excluding the component relating to the error term,

$$\sum_{r=1}^{J+1} i_r = R^2,$$

where R^2 is the familiar coefficient of determination of the regression for the income-generating equation. Thus the proportion of the inequality in $\ln(y)$ which is 'explained' by the variation in variable j is

$$p_j = i_j / R^2 .$$

Once the user specifies the set of explanatory variables, STATA will compute this explanatory decomposition.

6. Conclusions

The theme of this paper has been that measures of inequality should support attempts to explain the changes in inequality that have occurred. Compromises are often made necessary because of the nature of the data that are available. In such cases, it is important for the researcher to make the nature of these compromises clear. Most important, do the data refer to income or inequality and what is and is not included in these measures. The paper has argued for the use of income based measures, whenever possible, because most of our economic explanations for changes in inequality over time relate to income changes rather than expenditure changes.

It is also argued that the Generalized Entropy class of measures should be used, in addition to, but not instead of more conventional measures such as the Gini coefficient. While the Gini measure of inequality has serious technical disadvantages, it has the strength that it has been calculated for many countries over a long period and its general features are widely understood. Other measures, such as the Generalized Entropy class, can provide useful supplementary information.

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